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Formulation And Development Of A Mathematical Model For The Distribution Of A Parking Lot

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Abstract

This study presents the sections concerning the methodology of the research, introducing the problem, and then moving on to the preliminary location of the parking areas, made from the application of a mathematical procedure that models the behavior of the parking density of vehicles at peak hours of saturation, This procedure produces an algorithmic and analytical representation in R3 that finally defines the relative coordinates of strategic points that, according to the model to be developed, fully decongest the roads. A procedure is formulated and developed based on technical concepts of transportation engineering based on mathematical calculations and rational analysis that allows estimating the strategic location of parking areas to solve vehicular congestion. A difference of 0.910 in a range of 230, which shows a variation of 0.39%, sufficiently negligible considering maximum blocks of 100m, in which the absolute difference is less than 40cm.

Keywords: analysis; modeling; mathematics; quadratics; track; formula.

1. Introduction

The problem of vehicular saturation that leads to the active demand for tourism (Sebastián Truyols Mateu; (2011)) in conjunction with the lack of spaces for parking (Florencia Ucha (2015) in commercial sectors, as mentioned above, is frequent in the urban center and some villages of the municipality of Chinácota, where there are complications in mobility that slow the flow of them (Oña López, J; Oña López, R. (2018) given the decrease in the effective width of the road lanes due to the arbitrary and uncontrollable parking of tourist vehicles.

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Being a numerical description method, descriptive statistics (Sabadías, A. V. (1995) uses the number as a means to describe a set, which must be numerous, since statistical permanences do not occur in rare cases (Oré, C. G. (2011). It is not possible, therefore, to draw concrete and precise conclusions from statistical data (Veiga, N., Otero, L., & Torres, J. (2020). Descriptive statistics begins with the work of John Graunt (González-Hernández, I. J., Romero-Torres, R. I., Castillo-Leyva, A. E., Fernández-Amador, S. A., Juárez-García, J. A., & Santana-Robles, F. (2021) on the birth and death rates in London in the period from 1640 to 1661. Due to their origin, the terms used are proper to the field of demography (Vargas Sabadías, A., (1995).

It is convenient to make a distinction between what we call direct descriptive statistics, which aims to describe the relevant characteristics of a set of data, and of a set from the data of a subset of it (Pérez, I. M., Vázquez, L. S., & Pérez, E. M. (2019).

In addition, mathematical equations are used (Microsoft office, (2019). and “a partial derivative of a function of several variables, is the derivative with respect to each of those variables keeping the others as constants. Partial derivatives are useful in vector calculus, differential geometry, analytic functions, physics, mathematics, etc” (Wick, T. (2022).

“It is difficult to describe the derivative of such a function, since there are an infinite number of tangent lines at each point on its surface. Partial derivative is the act of choosing one of those lines and finding its slope. Generally, the lines of most interest are those that are parallel to the plane of the x-axis with z, and those that are parallel to the plane of the y-axis with z. A good way to find the values for those parallel lines is to treat the other variables as constants while leaving only one to vary” (Wikipedia Organization (2018).

Maximum point of a multivariable function (Universidad politécnica de Madrid 2019).

Let D be a region of the plane. Let f: D R →.

- It is said that f reaches its absolute maximum value M at a point “P = (x0, y0) ∈ D when.
- M = f (x0, y0) ≥ f (x, y) ∀ (x,y) ∈ D”.
- It is said that f has a relative maximum at a point “P = (x0, y0) ∈ D when f (x, y) ≥ f(x, y) ∀ (x,y) belonging to an environment of (x0 ,y0)”.
- It is said that f attains its absolute minimum value m at a point “P = (x0, y0) ∈ D when M=f (x0, y0) ≤ f (x, y) ∀ (x, y) ∈ D”.
- It is said that f has a relative minimum at a point “P = (x0, y0) ∈ D when f (x0, y0) ≤ f(x, y) ∀ (x, y)” belonging to an environment of (x0, y0).

2. Method

This research is carried out through the following steps:

- Model formulation and development
- Representations of vehicles on a roadway using interpolation functions.
- Creation of the surface equation
- Location of maximum demand points in R3

3. Results and discussion

3.3 Formulation and development of the model

This section develops the mathematical model for the formulation of this study, based on the technical concepts and mathematical tools described in the introduction of this document.

3.3.1. Representation of vehicles on a road (street or roadway) by means of interpolation functions:

- *Primary interpolation function (Arévalo-Ovalle, D., Bernal-Yermanos, M. Á., & Posada-Restrepo, J. A. (2021).*

For a set of equations *f1(x), f2(x), f3(x)... fn(x)* modeling a set of vectors (Morales Ramón, J., (2013). $\neg V_1$, $\neg V_2$, $\neg V_n$, obeying the street data record *k1, k2, k3... kn*; it is given that:

Equation 1. Polynomial interpolation function (Lazarus, C. C. (2018) of a street i.

$$fi(x) = C_G * X^G + C_{G-1} * X^{G-1} + C_{G-2} * X^{G-2} \ldots . C_{G-G} * X^{G-G}$$

Source: Own (2020)

Where:

fi(x): Interpolation function of road i.

x: Independent variable - street

Cj: Constant of the interpolation polynomial for term i: Subscript representing the function in question.

i: Subscript representing the function in question.

G: Degree of the function

Which can be expressed, in the form of a sequence, as follows:

Equation 2. Interpolation function expressed as a series.

$$fi(x) = \sum_{j=0}^{G} \left(C_j * X^j \right)$$

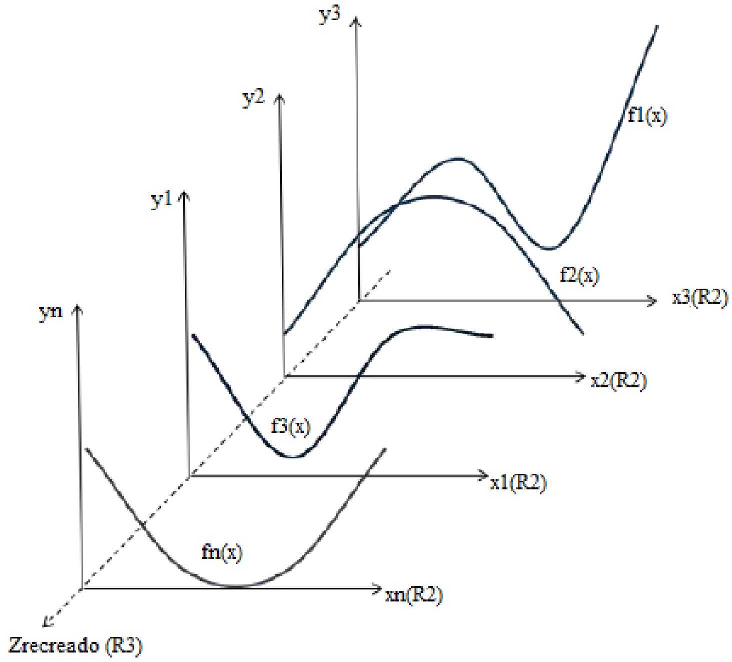
Source: Own (2020)

Each interpolation function that models the behavior of a direct log vector, will be called a primary interpolation function, and will comprise the form described above, where x represents the tracks in the direction orthogonal to the axis on which it is modeled.

- *Conjugation of primary interpolation functions to form a function in R3*

The primary interpolation functions are modeled in R2, their superposition on the orthogonal y-axis recreates a three-dimensional surface model, in R3.

Figure 1. Succession of functions in R2 for the recreation of the surface in R3.



Source: Own (2020)

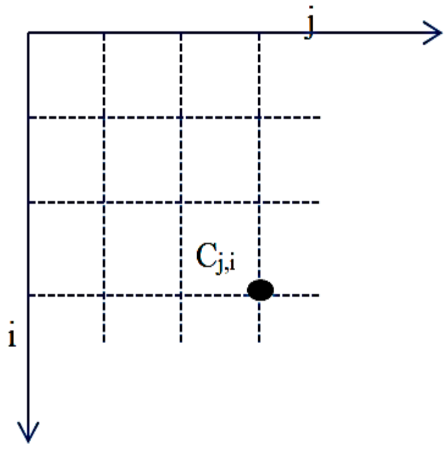
The conjugation of the n functions in R3 obeys to the recreation of a function in which the interpolation coefficients are a variable expressed as a sub equation of interpolation that crosses the axis and called secondary interpolation function (Reyes de la Cruz, R., (2012).

3.2. Creation of the surface equation

– *Secondary interpolation function*

For a set of primary functions fn(x), which vary as one traverses the axis orthogonal to its R2 planes, the mathematical expression that models the behavior of any trend at y = n, can be predicted with a secondary interpolation function in the interval $i \leq y \leq n$, as discussed, with a surface equation originated by expressing Cj as a function of fi and therefore as a function of y, as follows.

Figure 2. Secondary interpolation constant for a node (i,j) in the mesh.



Source: Own (2020)

Equation 3. Inclusion of the secondary interpolation constants as a function of the orthogonal axis.

$$C_{j,i} \rightarrow C_{j(y)} \Longrightarrow f_{i(x)} = \sum_{j=0}^{G} C_{j,i} * x^{j} = f_{(y=i,x)} = \sum_{j=0}^{G} C_{j(y=i)} * X^{j}$$

Source: Own (2020)

Obtaining, in this way:

Equation 4. Equation of surface area as a function of x and y. Source: Own (2020)

$$f_{(x,y)} = \sum_{j=0}^{G} C_{j(y)} * X^{j}$$

With:

Equation 5. Secondary interpolation constants as a series as a function of y.

$$C_{j(y)} = \sum_{j_2=0}^{G_2} C_{j2} * y^{j2}$$

Source: Own (2020)

Where:

G: Degree of the secondary polynomial equation.

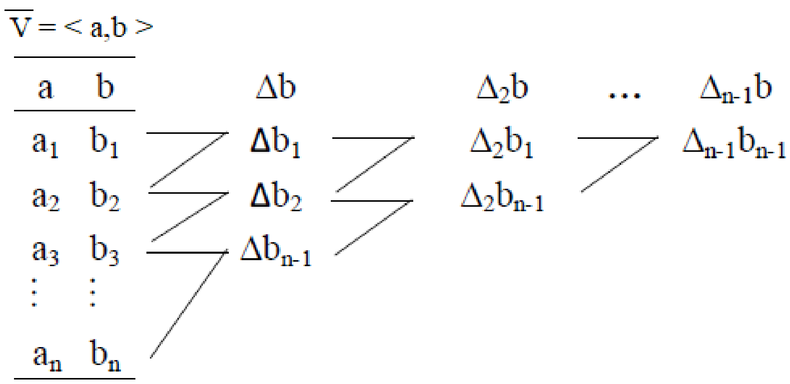
J2: Member of the secondary polynomial equation.

Cj2: Coefficient of the interpolation polynomial corresponding to member j2.

– *Generalization of the surface equation as a function of the number of Log data (Khan Academy (2019).*

It is to be noted that, for a set of n data (coordinates) of a vector $\vec{V}$, in the worst case, applying differences it is given that:

Figure 3. Convergence of the finite differences of the log vector.



Source: Own (2020)

The differentials of ¯V converge to ¯V at ¯V, i.e., at the n-1 difference, i.e., mathematically, G can be expressed as:

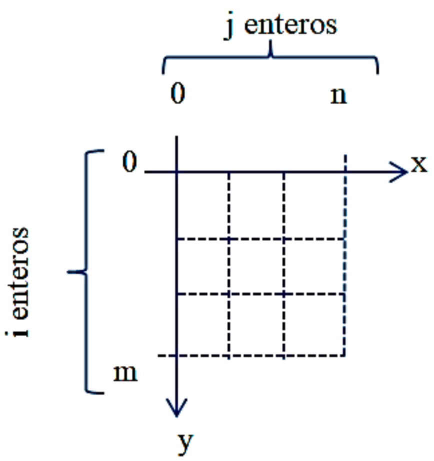
Equation 6. Degree of the interpolation polynomial as a function of the number of log data.

$$G = n-1$$

Source: Own (2020)

In a mesh where:

Figure 4. Nodal grid boundaries (amount of data in the record).



Source: Own (2020)

Then, it is generally true that:

Equation 7. Surface function expressed as a series, bi-variate (Stewart, J., (2012); represents the number of vehicles parked in the study area as a function of Cartesian coordinates.

$$f(x,y) = \sum_{j=0}^{n-1} \left[\left(\sum_{i=0}^{m-1} \{C_i * y^i \right) * X^j \right]$$

Source: Own (2020)

3.3. Location of points of maximum demand in R3:

– *Strategic location of parking lots*

The location of critical points of confluence of parking demand is, by definition, the strategic location of the centers of the action areas, i.e., parking lots.

As described in the introduction, the maximum point of a multivariable function, applying the concept of the spatial location of maximum values in R3, is nothing more than the formation of a system of equations where in the planes in R2, orthogonal to each other, and with the common straight line corresponding to the axis of the dependent variable, in this case: the data log, there is a maximum, that is, a maximum:

Equation 8. Posing the maximum point of a function in R3.

$$fx(x,y) = 0$$
$$fx(x,y) = 0$$

Source: Universidad politécnica de Madrid (2019).

Note that, if *f(x,y)* (Larson, Roland E., Edwards, B., 2010) is as presented in Equation 8, therefore, finding the coordinates of the point with the largest value in the surface equation within interval $D_x = [0, n]$; $D_y = [0, m]$one has:

Equation 9. Partial derivative with respect to x of the spatial function of parked vehicles.

$$fx(x,y) = \sum_{j=0}^{G} \left[j * C_{j(x)} * X^{j-1} \right] = \sum_{j=0}^{G_1} \left[j * \sum_{i=0}^{G_2} \left(C_{i * y^i} \right) * X^{j-1} \right] = 0$$

Source: Own (2020)

Equation 10. Partial derivative with respect to y of the spatial function of parked vehicles.

$$fy(x,y) = \sum_{j=0}^{G} \left[C'_{j(y)} * X^j \right] = \sum_{j=0}^{G_1} \left[\sum_{i=0}^{G_2} \left(i * C_{i * y^{i-1}} \right) * X^j \right] = 0$$

Fuente: Propia (2020)

This leads to the system of equations 2x2, as shown below:

Equation 11. 2x2 system of equations for the solution of the coordinates of the strategic location of the parking lots.

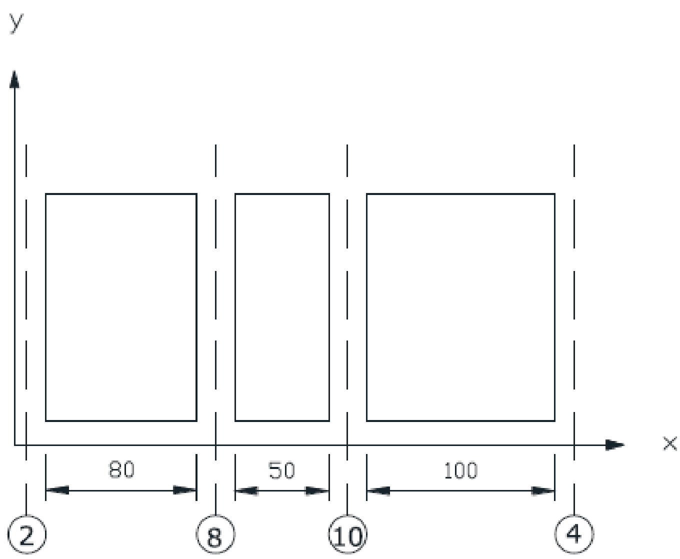
$$\begin{cases} \sum_{j=0}^{G_1}\left[j * \sum_{i=0}^{G_2}\left(C_i * y^i\right) * X^{j-1}\right] = 0 \\ \sum_{j=0}^{G_1}\left[\sum_{i=0}^{G_2}\left(i * C_i * y^{i-1}\right) * X^j\right] = 0 \end{cases}$$

Source: Own (2020)

– *Numerical demonstration of the equivalence for the calculation of maximums between real road lengths and nodal grid idealizations.*

Note that, at the beginning of the model approach, the lengths of the blocks (between streets and races) were idealized as equal, to calculate it as a grid where the values of the Cartesian coordinates of the plane are the numbering of the roads and not the real lengths of the same (Vector Calculus Journal, 2019), in order to model it by a nodal mesh that facilitates the distribution of vehicles parked at intersections as nodes, and therefore the subsequent calculations. In this section of the paper, the error or inaccuracy in the difference of applying the actual and ideal conditions is demonstrated analytically, in order to prove the efficiency of the method in terms of high accuracy of the results and considerable reduction of the level of difficulty of the calculations.

Figure 5. Plan drawing of an example with real lengths.



Source: Own (2020)

With vector $< x, y >$, in absolute coordinates:

Table 7. Vector $< x, y >$of the example.

X	Y
0	2
80	2
130	10
240	4

Source: Own (2020)

The interpolation polynomial is obtained:

Equation 12. Interpolation polynomial of the (real) numerical example.

$$y_{(x)} = -6{,}466x10^{-7} * x^3 - 0{,}0002488x^2 + 0{,}074x + 2$$

Source: Own (2020)

For which, with:

Equation 13. derivative of the interpolation polynomial of the (real) numerical example.

$$y'(x) = 0 = -1{,}94x10^{-6} * x^2 - 0{,}000498 * x + 0{,}074$$

Source: Own (2020)

Within range $0 \leq x \geq = 230$ then;

X= 105,446

For the same vector $< x, y >$, taking for the abscissa axis relative values defined by the variable z, then $< x, y > \leftarrow < z, y >$, where:

Table 8. Substitution of absolute values by relative coordinates of the axis.

X	Z
0	0
80	1
130	2
230	3

Source: Own (2020)

From which the interpolation equation is obtained:

Equation 14. Interpolation polynomial of the (ideal) numerical example.

$$y(z) = -0{,}667z^3 - 3{,}553x10^{-15}z^2 + 4{,}667z + 2$$

Source: Own (2020)

For which, with:

Equation 15. derivative of the interpolation polynomial of the (ideal) numerical example.

$$y'(z) = -2z^2 - 7{,}1054x10^{-15}z + 4{,}667 = 0$$

Source: Own (2020)

Within the range 0<=z<=3, then:

Z=1,527

Note that the value of z refers to a fraction of *0.527* after *1*, i.e., *52.7%* of the length of the section following node 1 plus the cumulative length up to node 1, so that:

Equation 16. Calculation of the approximate value of x for the idealized model with z-substitution.

$$x\ = (z = 1{,}527) = x(z = 1) + 0{,}527[x(z = 1)] = 80 + 0{,}527(50) = 106{,}376$$

Source: Own (2020)

With absolute values: x=105,446
With relative values: x =106,376

5. Conclusions

A difference of 0.910 is presented in a range of 230, which shows a variation of 0.39%, sufficiently negligible considering maximum blocks of 100m, in which the absolute difference is less than 40cm, which is much less than the average width of a typical property, so it is concluded that the hypothesis of formulation of the model to adjust to a uniform mesh in relative grids absolute values of continuous variable sections is widely acceptable.

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